Brownian motion and Stochastic Calculus
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Assignment 4

Exercise 1

Show the results stated in Remark 2.1.2 in the lecture notes.

Exercise 2

Prove Proposition 2.2.3.

Exercise 3

Prove Proposition 2.2.4.

Exercise 4

A continuous time stochastic process is called a *Brownian Bridge* if it is a Gaussian process with mean 0 and covariance function $s(1-t)$, $s < t$. Let W be a Brownian motion and consider the process $X = (X_t)_{0\leq t\leq 1}$ defined by $X_t =: W_t - tW_1$.

1) Show that X is a Brownian Bridge, and that X does **not** have independent increments.

2) Show that if $(X_n)_{n\in\mathbb{N}}$ is a Gaussian process indexed by $\mathbb{N}$ and converges in probability to a random variable X as n goes to infinity, then it converges also in $\mathbb{L}^2(\mathbb{R}, \mathcal{F}, \mathbb{P})$ to X.